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"LUNAR SPRING-BOARD" EFFECT

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ABSTRACT. The problem of minimizing the sum of velocity impulses to acquire a given velocity starting from a low orbit around the Earth and at a great distance from the Earth-Moon distance. The "spring-board effect" of the Moon is used (passing near the Moon).

In this case the problem can be decomposed into three phases if the simplifying assumption of a lunar "activity sphere" is used. These are: Earth-Moon transfer, hyperbolic passage near the Moon, and acquisition of the particle velocity at infinity. The method of optimizing each of these three phases is discussed.

The simple case of "grazing" flyby is studied (which should not be far from the global optimum). This leads to a velocity impluse saving of 158 m/s at the most, compared with direct ascent.

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I. INTRODUCTION

Considering the cost of space experiments, the optimization of orbital maneuvers will obviously be of interest.

For the majority of present-day rockets, the minimum propellent mass consumption results by using "minimum characteristic velocity transfers[(1)]". This is the case of chemical rockets and the case of the majority of nuclear-electric propulsion systems.

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*Numbers in the margin indicate pagination in the original foreign text.

(1)The characteristic velocity is the integral of the thrust acceleration.

It is from this point of view that we will now study transfers starting from a low orbit around the Earth to a given hyperbolic velocity at the limit of the "activity sphere" of the Earth. We will try to do this by taking advantage as much as possible of the gravitational field of the Moon.

A simple case very close to the true optimum will be studied numerically.

II. NOTATION

A angle of $\vec{V_1}, \vec{V_2}$ $(0^0 \leqslant A \leqslant 180^0)$.

A_1 semi-deviation angle for a hyperbola which grazes the Moon with the velocity at infinity of $V_1 \left(A_1 = \arc \sin \frac{L'^2}{L'^2 + 2V} \right)$ (Figure 1).

α angle of $\vec{V_E}, \vec{V_L}$ $(0 \leqslant \alpha \leqslant 89^0\ 3'\ 2,\ 15'')$.

C_d characteristic velocity of "direct ascent".

C_α characteristic velocity of the transfer with passage near the Moon.

E savings due to the lunar spring-board effect ($E = C_d - C_\alpha$).

e relative saving $\left(e = \frac{C_d - C_\alpha}{C_d} \right)$

H hyperbola of the velocity hodagraph $\vec{V}_E$.

L libration rate of the Earth at the distance R.

L' libration rate of the Moon on the Earth.

L_S libration rate of the Earth at the distance r

$$\left(= L \sqrt{\frac{R}{r}} \right).$$

R radius of the low orbit O_B.

r radius of the Lunar orbit.

S Moon (satellite)

T Earth.

U_1 escape velocity at the surface of the Moon starting from the velocity V_1 at infinity of the Moon $\left(U_1 = \sqrt{V_1^2 + L'^2} \right)$.

$\vec{V}_E$ velocity at entry into the sphere of influence of the Moon.

$\vec{V}_S$ velocity at the exit from the lunar sphere of influence.

$\vec{V}_1 = \vec{V}_E - \vec{V}_L$ (Figures 1 and 3).

$\vec{V}_2 = \vec{V}_S - \vec{V}_L$.

V_C circular velocity at distance R from the center of the Earth $\left(= L/\sqrt{2}\right)$.
V_L rotation rate of the Moon around the Earth (in circular orbit) $= L_s/\sqrt{2}$.
V_∞ velocity required at infinity for the Earth-Moon system.
α angle of $\vec{V_E}, \vec{V_L}$, $(0 \leqslant \alpha \leqslant 89° \, 3' \, 2, 15'')$.
μ gravitational constant of the planet (product of its mass by the Newtonian constant).

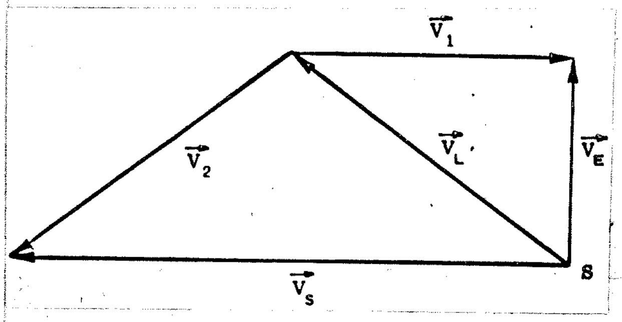

Figure 1. Decomposition of the velocities.

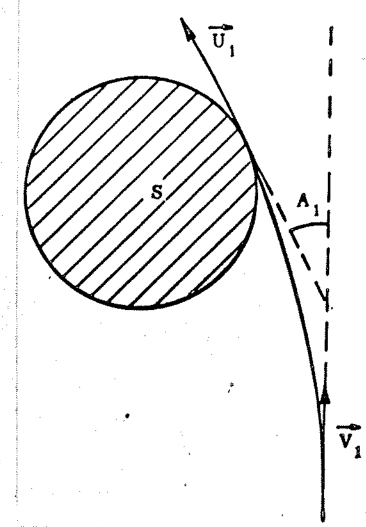

Figure 2. Angle A_1 and velocity μ_1.

III. DESCRIPTION OF THE THEORETICAL PROBLEM

The problem of the lunar spring-board effect is a particular case of the problem of three bodies. This means that the complete analysis is difficult. This is why we will analyze a simplified and closely related problem in which the Earth is a spherical attracting body of given mass and radius and the Moon maintains its libration rate on the ground (2 372 m/s). However, it is assumed that it has an infinitesimally small radius and mass. In addition we will assume that it describes a circular orbit around the Earth with a rotation rate V_L (1 017 m/s).

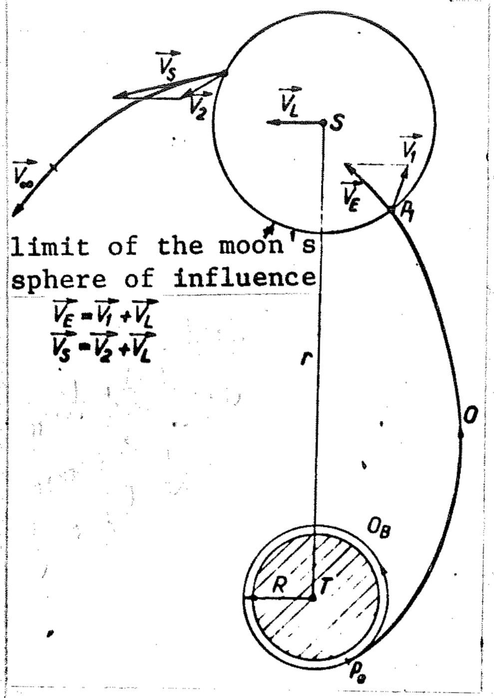

Figure 3. Phases of the mission.

Under these conditions the sphere of influence of the Moon is infinitesimally small with respect to the Earth and infinitely large with respect to the Moon. /65

Although the error committed is important as far as the duration of the trajectories are concerned (a parameter which is not of interest here), it remains very small as far as the velocities and the fuel consumption is concerned.

The problem consists of three phases (Figure 3):

-- Phase I: terrestial transfer phase from a low orbit O_B, which is circular, around the Earth and goes to the Moon;

-- Phase II: lunar phase with a hyperbolic path between the entry and departure from the lunar sphere of influence;

-- Phase III: terrestial phase in which the required velocity at infinity is acquired.

IV. STUDY OF THE THREE PHASES

The study of the lunar phase which consists of a study of optimum transfers between hyperbolic velocities has already been performed [2 - 4 and 7]. However, we will describe the fundamental ideas which are involved in the last two phases. This will make it possible to take up the numerical simplified study which will be presented.

Let us consider (Figure 4) a transfer orbit O which is tangential to the low orbit O_B at the point P_o and which passes through the point S with an entry

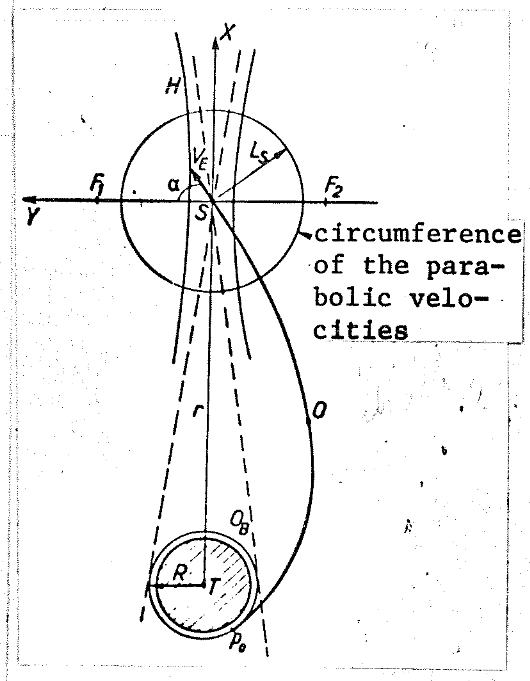

Figure 4. Hodograph hyperbola.

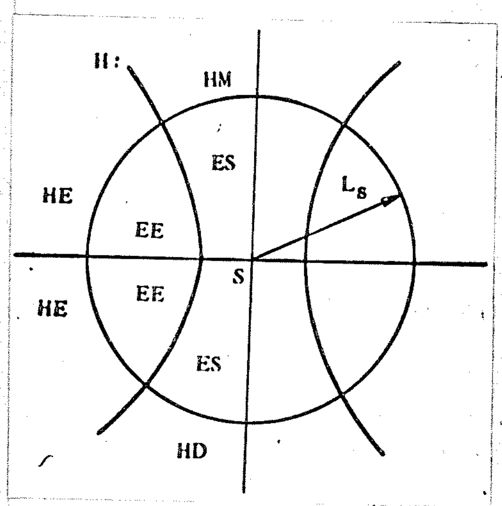

Figure 5. Regions of entry velocities.

velocity V_E upon entering the lunar sphere of influence (with infinitesimally small radius). Let X, Y be the components of V_E with respect to the system of axes SXY. The velocity hodograph V_E for the orbits O is a hyperbola H which has the equation

$$Y^2 r\left(r^2 - R^2\right) - X^2 r R^2 = 2\mu R (r - R). \qquad (1)$$

The asymptotes of H are the tangents to the low orbit O_B drawn from the point S. It can be seen that the eccentricity of H is $\frac{r}{R}$.

By considering in Figure 5 the hodograph hyperbola and the circumference with the center S and radius L_S, which corresponds to parabolic entry velocities, it is possible to obtain the following partition of the ensemble entry velocities V_E.

- ES: region of elliptic velocities $\vec{V}_E$ which intersect the Earth.
- EE: region of external elliptic velocities $\vec{V}_E$.
- HM: region of hyperbolic velocities $\vec{V}_E$ which intersect during ascent.
- HD: region of hyperbolic velocities $\vec{V}_E$ which intersect during descent.
- HE: region of hyperbolic velocities $\vec{V}_E$ which are exterior.

Using this partition, it is possible to study the various types of optimum transfers in Phase I and to prove that the optimum case only consists of two impulses at the most.

For the lunar Phase II, there are twelve types of optimum transfers which go from $\vec{V}_1$ to $\vec{V}_2$ (and do not intersect the ground) [4, 7]. The transfers consist of two impulses at the most, except in the P.N.P. case (which includes the parabolic level) where it consists of six impulses. The case which will be of interest in the numerical study is the one which is referred to as "free transfer" in [4, 7]. In this case $V_1 = V_2$ and $A \leq 2\ A_1$ where $\vec{V}_1$ and $\vec{V}_2$ are the asymptotic velocities of an identical hyperbolic orbit which does not intersect the Moon (Figure 6).

For the Phase III from $\vec{V}_S$ (velocity at the exit from the lunar sphere of influence) to $\vec{V}_\infty$ it is possible to carry out a study which is similar to the one for Phase I. One can prove that the optimum case consists of only three impulses at the most.

The overall optimum depends on the exit velocity $\vec{V}_\infty$. It is possible to show that if V_∞ is large, the savings due to the lunar spring-board effect is always 92 m/s, the existing velocity $\vec{V}_S$ is parabolic, and the parabola-hyperbolic transfer of Phase III is the classical one [5].

If, on the other hand, V_∞ is small (less than 559 m/s), the optimum transfer is the one shown in Figure 7 with only one tangential impulse at P_o and a free pass during the lunar phase.

Finally if V_∞ has average size the optimum transfer is bi-impulsive (Figure 8) with one tangential impulse at P_o and an impulse at I_2 in the vicinity /66
of the Moon. The lunar phase is of the RF type: grazing plus an impulse at a finite distance [4, 7]. The latter case is the one which was studied numerically in an approximate way by replacing the lunar phase of type RF by a free grazing transfer.

V. NUMERICAL STUDY OF A SIMPLE CASE NEAR THE OPTIMUM OF THE THREE PHASES

The numerical values of the principal parameters which occur in the calculation are:

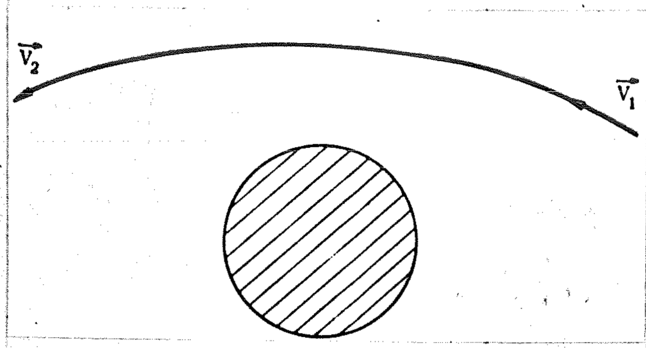

Figure 6. Free pass.

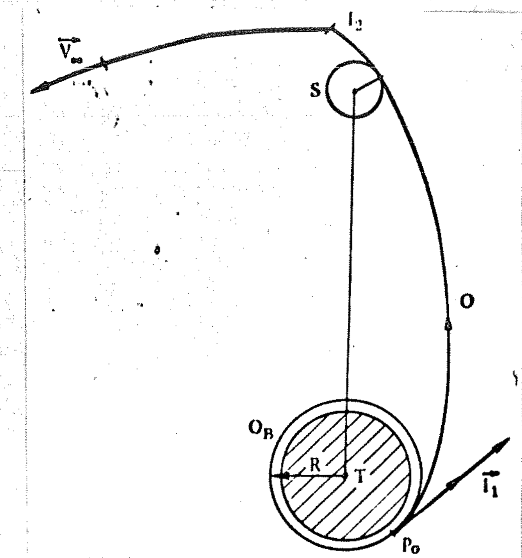

Figure 8. Optimum ascent
(case: 559 m/s < V_∞ < 3.5 km/s).

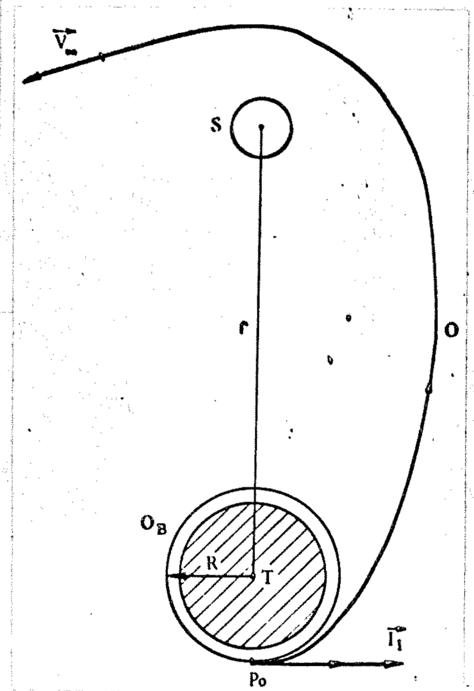

Figure 7. Optimum ascent
(case V_∞ < 559 m/s).

$R = 6\,378{,}165$ km $\qquad r = 384\,400$ km

$V_L = 1{,}017$ km/s

$L = 11{,}1859$ km/s $\qquad L/\sqrt{2} = 7{,}909$ km/s

$L' = 2{,}372$ km/s

$$K = \frac{R}{r} = 0{,}01659$$

$$C_o = L\left(\sqrt{\frac{r}{R+r}} - \frac{\sqrt{2}}{2}\right) = 3{,}185 \text{ km/s}$$

$$C_{oo} = C^d\ (V_\infty = 0) = 3{,}277 \text{ km/s}.$$

V.1 Direct Ascent

If we neglect the possibility of "the lunar spring board effect", the optimum transfer which leads from the low orbit O_B at the velocity $\vec{V}_\infty$ is the "direct ascent" (Figure 9) which has one single tangential impulse I_1 at P_o:

$$\vec{I}_1 = \vec{U} - \vec{V}_c.$$

The characteristic velocity of the "direct ascent" is

$$\boxed{C_d = \left|\vec{U} - \vec{V}_c\right| = \sqrt{L^2 + V_\infty^2} - L/\sqrt{2}},$$

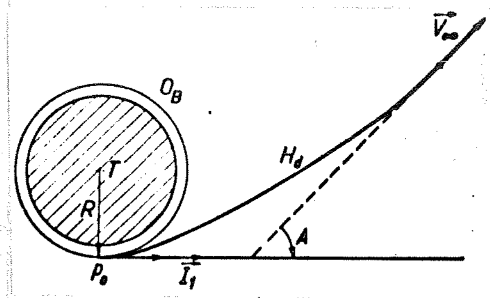

Figure 9. Direct ascent.

The quantity C_d is shown as a function of V_∞ in Figure 12.

The values which are most useful in the following are:

$$V_\infty = 0 \Rightarrow C_d = 3{,}277 \text{ km/s};$$
$$V_\infty = 0{,}559 \text{ km/s} \Rightarrow C_d = 3{,}291 \text{ km/s}.$$

V.2 Acquisition of the Velocity at Infinity from the Low Orbit.

Two cases may be distinguished which are characterized by the value of V_∞:

First Case $\qquad 0 \leqslant V_\infty \leqslant 0{,}559$ km/s.

The transfer is shown in Figure 10.

Phase I: a tangential impulse at P_0 leads to the orbit O, the apogee of which is along the lunar orbit.

The impulse is therefore:

$$I_1 = L\left[\sqrt{\frac{r}{r+R}} - \frac{\sqrt{2}}{2}\right] = 3{,}185 \text{ km/s}.$$

and

$$|\vec{V}_E| = \text{apogee velocity at } O = L_S\sqrt{\frac{R}{r+R}} = 0{,}184 \text{ km/s}.$$

Phase II: This is a hyperbolic transfer of the free type [4, 7].

Phase III: There is no new impulse; therefore

$$C_a = I_1 = 3{,}185 \text{ km/s}.$$

Depending on the lunar passage distance, V_∞ can be obtained from zero to 0.559 km/s. This latter case corresponds to a grazing path. The angle between $\vec{V}_1$ and $\vec{V}_2$ is therefore 112.8° (Figure 10).

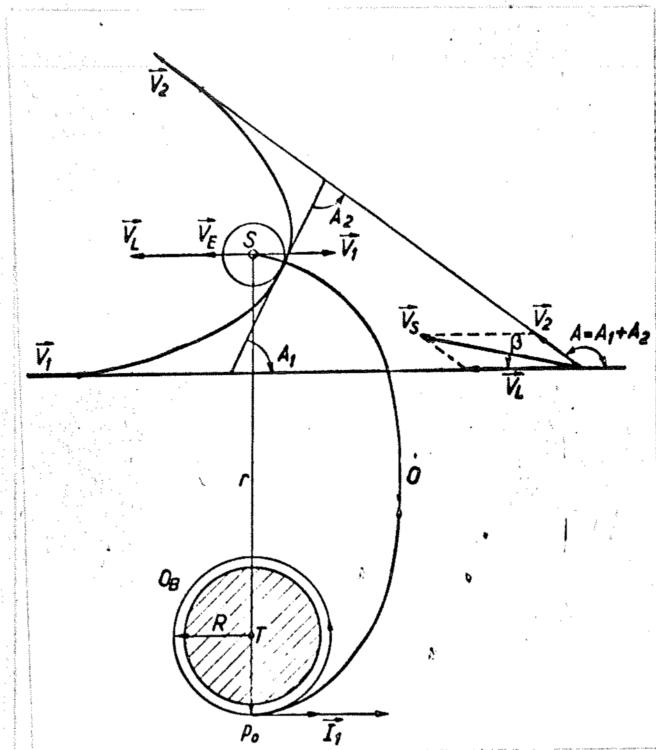

Figure 10. Simple models for $\vec{V}_E \equiv$ apogee velocity of the orbit O.

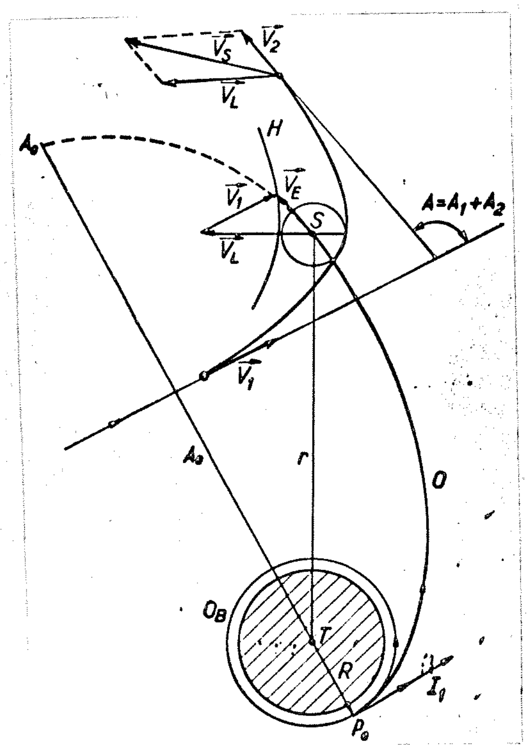

Figure 11. Simple model for $\vec{V}_E$ for the hodography hyperbola H.

The characteristic velocity of these transfers is therefore always equal to I_1 (3,185 km/s), and the saving obtained with respect to "the direct ascent" (Figure 9) is therefore /67

$$E = \sqrt{V_\infty^2 + L^2} - L/\sqrt{2} - I_1 .$$

It increases from 92 m/s to 106 m/s when V_∞ increases from 0 to 559 m/s.

Second Case

The transfer is shown in Figure 11.

Phase I: still consists of a tangential impulse at P_0 which leads to the orbit O, the apogee of which no longer is the lunar orbit but beyond it.

The impulse is therefore

$$I_1 = L\left(\sqrt{\frac{A_0}{A_0 + R}} - \frac{\sqrt{2}}{2}\right)$$

The arrival velocity V_E upon entry into the lunar sphere of influence is

$$V_E = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{A_0 + R}\right)}$$

which must be the same as the one calculated from the hodograph hyperbola (Figure 4).

$$V_E^2 = L_S^2 \frac{R(r - R)}{r^2 \cos^2 \alpha - R^2}.$$

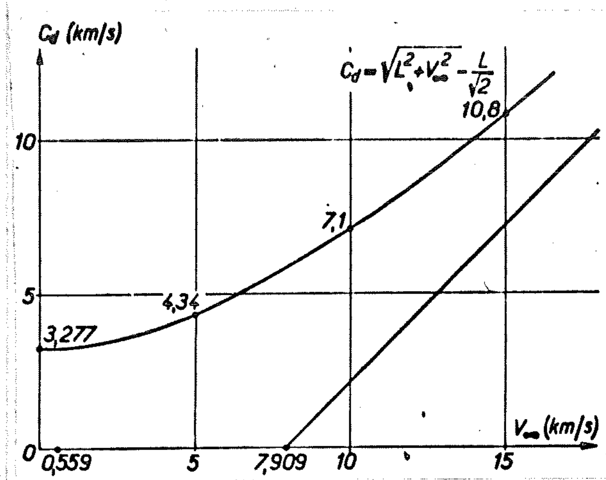

Figure 12. Characteristic velocity of the direct ascent.

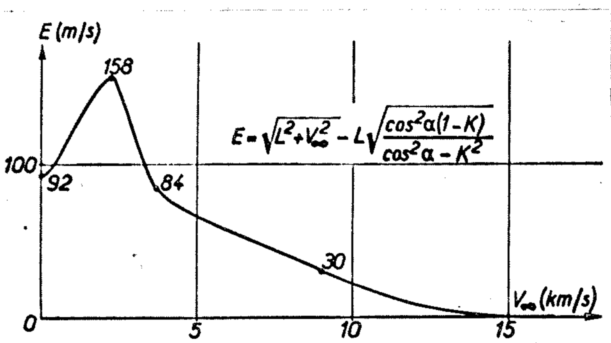

Figure 13. Characteristic velocity savings due to the lunar spring-board effect.

We obtain the following impulse from these three equations:

$$I_1 = L\left[\sqrt{\frac{(1-K)\cos^2\alpha}{\cos^2\alpha - K^2}} - \frac{\sqrt{2}}{2}\right].$$

Phase II:

This is a hyperbolic path of the free and grazing types [4, 7].

Phase III:

There are no new impulses; therefor $C_\alpha = I_1$.

Depending on the impulse I_1, it is possible to obtain V_∞ from 0.559 km/s up to infinity which corresponds to $\cos\alpha = \frac{R}{r}$; therefore α = 89° 3'2, 25" (limiting angle between $\vec{V}_E$ and $\vec{V}_L$).

The angle between $\vec{V}_1$ and $\vec{V}_2$ (Figure 11) varies between 112.8° and 0°.

The characteristic velocity I_1 of the transfers considered as a function of V_∞ is given by a very complicated expression.

We have studied the saving function $E = C_d - I_1$ and it is shown in Figure 13. The maximum of E is 158 m/s which is obtained for α = 84°.

The function $e = \frac{C_d - C_\alpha}{C_d}$ = is the relative saving and is shown in Figure 14.

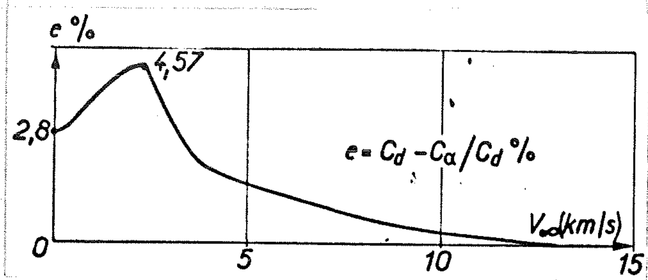

Figure 14. Percentage of saving.

VI. CONCLUSION

Use of the "lunar spring-board effect" leads to considerable characteristic velocity savings which are always larger than or equal to 92 m/s. In particular, for exit velocities from the terrestrial activity sphere less than 3.4 km/s, it leads to a saving between 92 m/s and 158 m/s, depending on V_∞, if the simplified model is used with one single tangential impulse near the Earth and a "free path" near the Moon.

It will be interesting to study a more complete model with two impulses, one near the Earth and the other near the Moon. This model would probably lead to a maximum savings of about 200 m/s.

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